

Théorie de la synchronisation - Equation de Duffing

Etude du régime transitoire - Oscillateur en forte avance sur l'excitation

Caractéristiques du système

$$T := 0.2 \cdot s \quad \omega_0 := \frac{2 \cdot \pi}{T} \quad J := 8 \cdot 10^{-7} \cdot kg \cdot m^2 \quad q_0 := 270 \cdot deg$$

Frottement visqueux

$$\eta := 0.002 \quad C := 2 \cdot J \cdot \eta \cdot \omega_0 \quad F_{v_max} := C \cdot \omega_0 \cdot q_0 \quad \lambda := \frac{F_{v_max}}{J \cdot \omega_0^2} \quad h := 2 \cdot \frac{\eta}{\lambda}$$

$$F_{v_max} = 0.015 N \cdot mm \quad \lambda = 0.019 \quad h = 0.212$$

Frottement quadratique

$$B := 0.06 \cdot F_{v_max} \quad \beta_1 := \frac{B}{\lambda \cdot J \cdot \omega_0^2} \quad \beta_1 = 0.06 \quad F_{q_max} := B \cdot q_0^3 \quad F_{q_max} = 0.093 N \cdot mm$$

Régimes transitoires vers un foyer unique de grande amplitude

Excitation harmonique

$$A_c := \frac{8 \cdot h^3}{3 \sqrt{3}} \quad A_c = 0.015 \quad \boxed{A := 2 \cdot A_c} \quad A = 0.029$$

$$a_1 := \sqrt{\frac{4 \cdot A}{3 \cdot \beta_1}} \quad F_{harm} := a_1 \cdot (\lambda \cdot J \cdot \omega_0^2) \quad F_{harm} = 1.203 \times 10^{-5} N \cdot m$$

$$n := 500 \quad i := 0..n \quad x_0 := 0 \quad x_1 := 2 \cdot \pi \quad \Delta x := \frac{x_1 - x_0}{n} \quad x_i := x_0 + i \cdot \Delta x$$

$$Y_i := \frac{a_1}{h} \cdot \sin(x_i) \cdot (0 < x_i < \pi) \quad \varepsilon := 2 \cdot h \quad \sqrt{3} = 1.732 \quad \boxed{\varepsilon = 0.424}$$

$$X := \left[\begin{array}{l} \text{for } i \in 0..n \\ \left| \begin{array}{l} Z \leftarrow \text{polyracines} \left(\left(-a_1 \cdot \cos(x_i) \quad \varepsilon \quad 0 \quad \frac{-3}{4} \cdot \beta_1 \right)^T \right) \\ \text{for } j \in 0..2 \\ X_{i,j} \leftarrow Z_j \cdot (Im(Z_j) = 0) \cdot (Re(Z_j) > 0) \end{array} \right. \\ X \end{array} \right]$$

$$v := \left(-A \quad \varepsilon^2 + h^2 \quad -2 \cdot \varepsilon \quad 1 \right)^T \quad Z := \text{polyracines}(v) \quad y_s := \sqrt{\frac{4 \cdot Z}{3 \cdot \beta_1}} \quad x_s := \arccos \left[\frac{1}{a_1} \cdot \left(\varepsilon \cdot y_s - \frac{3}{4} \cdot \beta_1 \cdot y_s^3 \right) \right]$$

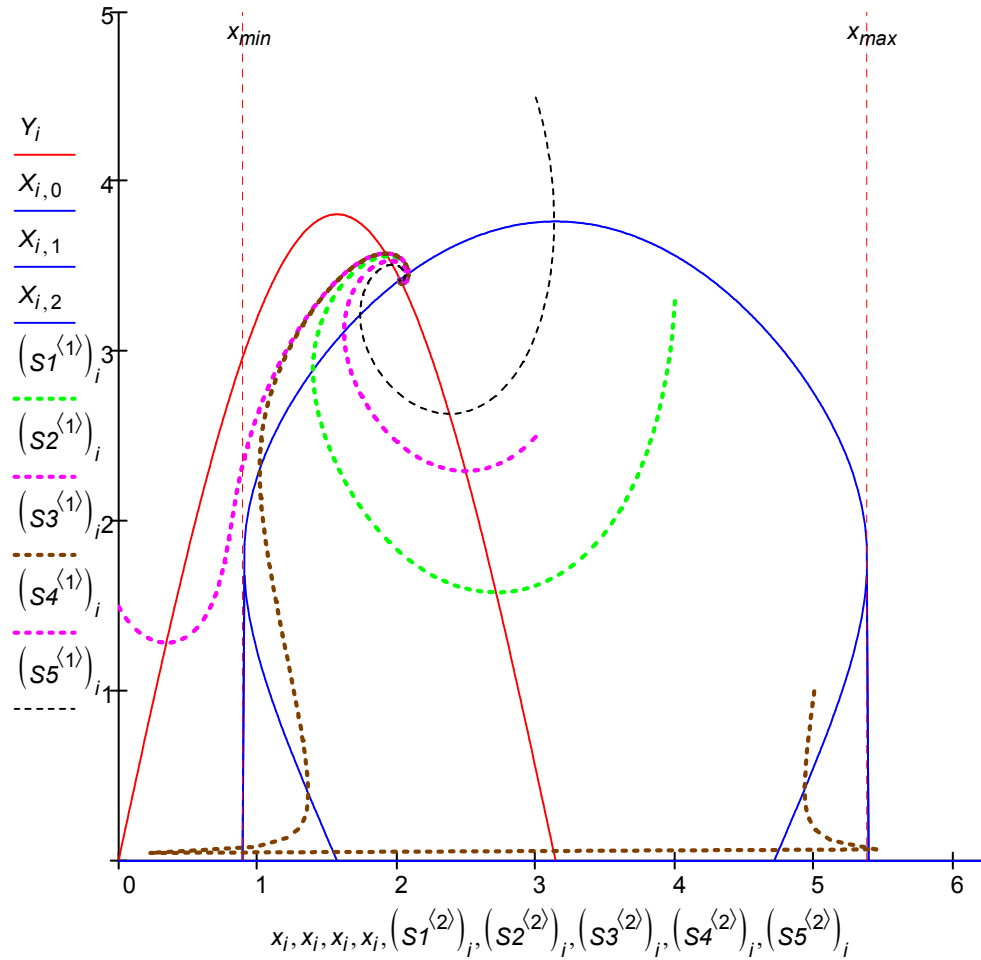
$$\frac{Z}{h} = \begin{pmatrix} 0.756 - 0.816i \\ 0.756 + 0.816i \\ 2.488 \end{pmatrix} \quad y_s = \begin{pmatrix} 2.099 - 0.917i \\ 2.099 + 0.917i \\ 3.425 \end{pmatrix} \quad x_s = \begin{pmatrix} 0.559 - 0.28i \\ 0.559 + 0.28i \\ 2.025 \end{pmatrix} \quad x_{min} := \arccos \left(\frac{4}{9} \cdot \frac{\varepsilon}{a_1} \cdot \sqrt{\frac{\varepsilon}{\beta_1}} \right)$$

$$x_{max} := 2 \cdot \pi - x_{min}$$

$$x_{min} = 0.902 \quad x_{max} = 5.382$$

$$Y0 := \begin{pmatrix} 3.3 \\ 4 \end{pmatrix} \quad D(t, Y) := \begin{pmatrix} \left(\frac{-h}{2} \cdot Y_0 + \frac{a_1}{2} \cdot \sin(Y_1) \right) \lambda \\ \lambda \cdot \left[\frac{-\varepsilon}{2} + \frac{3}{8} \cdot \beta_1 \cdot (Y_0)^2 + \frac{a_1}{2 \cdot Y_0} \cdot \cos(Y_1) \right] \end{pmatrix}$$

$t_f := 4000$	$S1 := rkfixe(Y0, 0, t_f, n, D)$	$S1^{(2)} := mod(S1^{(2)}, 2 \cdot \pi)$	
$Y0 := \begin{pmatrix} 1.5 \\ 0 \end{pmatrix}$	$S2 := rkfixe(Y0, 0, t_f, n, D)$	$Y0 := \begin{pmatrix} 1 \\ 5 \end{pmatrix}$	$S3 := rkfixe(Y0, 0, t_f, n, D)$
	$S2^{(2)} := mod(S2^{(2)}, 2 \cdot \pi)$		$S3^{(2)} := mod(S3^{(2)}, 2 \cdot \pi)$
$Y0 := \begin{pmatrix} 2.5 \\ 3 \end{pmatrix}$	$S4 := rkfixe(Y0, 0, t_f, n, D)$	$Y0 := \begin{pmatrix} 4.5 \\ 3 \end{pmatrix}$	$S5 := rkfixe(Y0, 0, t_f, n, D)$
	$S4^{(2)} := mod(S4^{(2)}, 2 \cdot \pi)$		$S5^{(2)} := mod(S5^{(2)}, 2 \cdot \pi)$



Régimes transitoires vers un foyer stable et un noeud structurellement instable

Excitation harmonique

$$A := 2 \cdot A_c$$

$$A = 0.029$$

$$\varepsilon := 2.4402 \cdot h$$

$$\varepsilon = 0.518$$

$$a_1 := \sqrt{\frac{4 \cdot A}{3 \cdot \beta_1}}$$

$$F_{harm} := a_1 \cdot (\lambda \cdot J \cdot \omega_0^2)$$

$$F_{harm} = 1.203 \times 10^{-5} \text{ N.m}$$

$$\sqrt{3} = 1.732$$

$$Y_i := \frac{a_1}{h} \cdot \sin(x_i) \cdot (0 < x_i < \pi)$$

$$X := \left[\begin{array}{l} \text{for } i \in 0..n \\ \left| \begin{array}{l} Z \leftarrow \text{polyracines} \left(\left(-a_1 \cdot \cos(x_i) \quad \varepsilon \quad 0 \quad -\frac{3}{4} \cdot \beta_1 \right)^T \right) \\ \text{for } j \in 0..2 \\ X_{i,j} \leftarrow Z_j \cdot (Im(Z_j) = 0) \cdot (Re(Z_j) > 0) \end{array} \right. \\ X \end{array} \right]$$

$$v := \begin{pmatrix} -A & \varepsilon^2 + h^2 & -2 \cdot \varepsilon & 1 \end{pmatrix}^T \quad Z := \text{polyracines}(v) \quad y_s := \sqrt{\frac{4 \cdot Z}{3 \cdot \beta_1}} \quad x_s := \arccos \left[\frac{1}{a_1} \cdot \left(\varepsilon \cdot y_s - \frac{3}{4} \cdot \beta_1 \cdot y_s^3 \right) \right]$$

$$\frac{Z}{h} = \begin{pmatrix} 1.046 \\ 1.062 \\ 2.773 \end{pmatrix}$$

$$y_s = \begin{pmatrix} 2.221 \\ 2.238 \\ 3.616 \end{pmatrix}$$

$$x_s = \begin{pmatrix} 0.622 \\ 0.628 \\ 1.892 \end{pmatrix}$$

$$x_{min} := \arccos \left(\frac{4}{9} \cdot \frac{\varepsilon}{a_1} \cdot \sqrt{\frac{\varepsilon}{\beta_1}} \right)$$

$$x_{max} := 2 \cdot \pi - x_{min}$$

$$x_{min} = 0.581 \quad x_{max} = 5.703$$

$$Y0 := \begin{pmatrix} 3.3 \\ 4 \end{pmatrix} \quad D(t, Y) := \begin{bmatrix} \left(\frac{-h}{2} \cdot Y_0 + \frac{a_1}{2} \cdot \sin(Y_1) \right) \lambda \\ \lambda \cdot \left[\frac{-\varepsilon}{2} + \frac{3}{8} \cdot \beta_1 \cdot (Y_0)^2 + \frac{a_1}{2 \cdot Y_0} \cdot \cos(Y_1) \right] \end{bmatrix}$$

$$t_f := 4000$$

$$S1 := \text{rkfixe}(Y0, 0, t_f, n, D)$$

$$S1^{(2)} := \text{mod}(S1^{(2)}, 2 \cdot \pi)$$

$$Y0 := \begin{pmatrix} 1.5 \\ 0 \end{pmatrix}$$

$$S2 := \text{rkfixe}(Y0, 0, t_f, n, D)$$

$$Y0 := \begin{pmatrix} 2 \\ 5 \end{pmatrix}$$

$$S3 := \text{rkfixe}(Y0, 0, t_f, n, D)$$

$$S2^{(2)} := \text{mod}(S2^{(2)}, 2 \cdot \pi)$$

$$S3^{(2)} := \text{mod}(S3^{(2)}, 2 \cdot \pi)$$

$$Y0 := \begin{pmatrix} 2.7 \\ 3 \end{pmatrix}$$

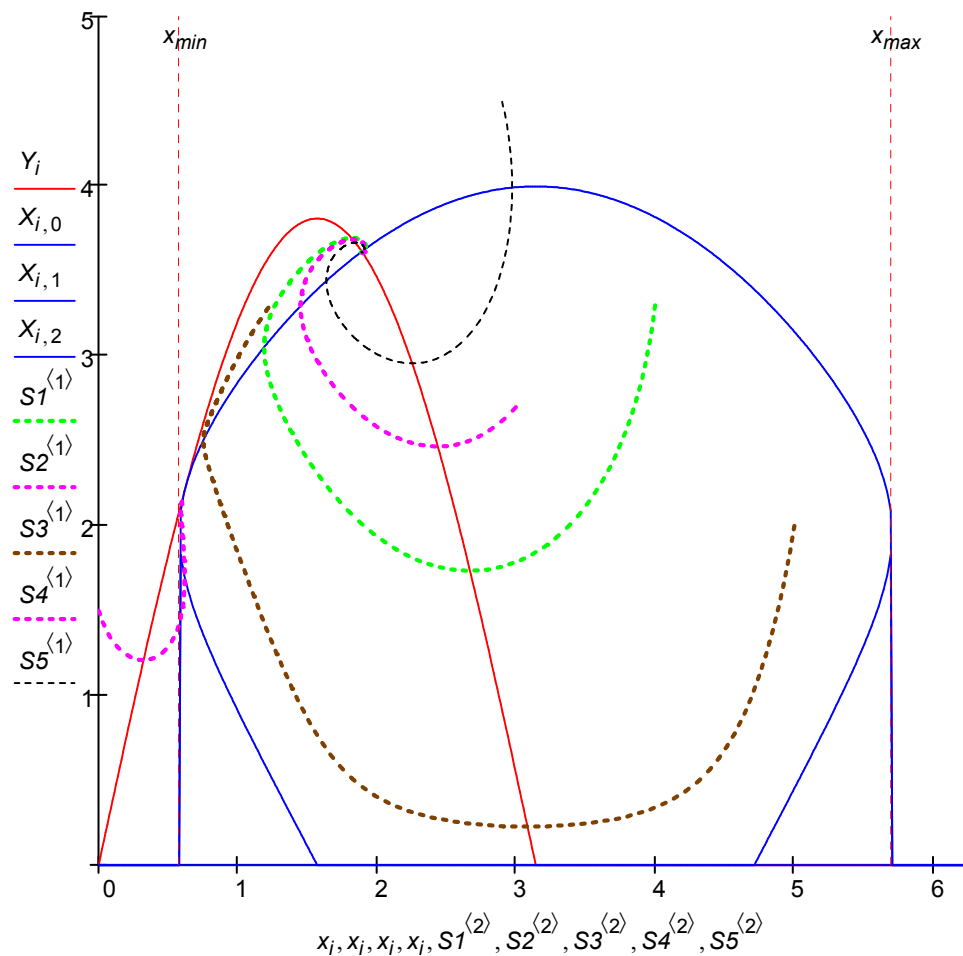
$$S4 := \text{rkfixe}(Y0, 0, t_f, n, D)$$

$$Y0 := \begin{pmatrix} 4.5 \\ 2.9 \end{pmatrix}$$

$$S5 := \text{rkfixe}(Y0, 0, t_f, n, D)$$

$$S4^{(2)} := \text{mod}(S4^{(2)}, 2 \cdot \pi)$$

$$S5^{(2)} := \text{mod}(S5^{(2)}, 2 \cdot \pi)$$



Régimes transitoires vers deux foyers stables

Excitation harmonique

$$A := 2 \cdot A_c$$

$$A = 0.029$$

$$\varepsilon := 2.8 \cdot h$$

$$\varepsilon = 0.594$$

$$a_1 := \sqrt{\frac{4 \cdot A}{3 \cdot \beta_1}}$$

$$F_{\text{harm}} := a_1 \cdot (\lambda \cdot J \cdot \omega_0^2)$$

$$F_{\text{harm}} = 1.203 \times 10^{-5} \text{ N} \cdot \text{m}$$

$$\sqrt{3} = 1.732$$

$$Y_i := \frac{a_1}{h} \cdot \sin(x_i) \cdot (0 < x_i < \pi)$$

$$X := \left[\begin{array}{l} \text{for } i \in 0..n \\ \left| \begin{array}{l} Z \leftarrow \text{polyracines} \left(\left(-a_1 \cdot \cos(x_i) \quad \varepsilon \quad 0 \quad \frac{-3}{4} \cdot \beta_1 \right)^T \right) \\ \text{for } j \in 0..2 \\ X_{i,j} \leftarrow Z_j \cdot (Im(Z_j) = 0) \cdot (Re(Z_j) > 0) \end{array} \right. \\ X \end{array} \right]$$

$$v := \begin{pmatrix} -A & \varepsilon^2 + h^2 & -2 \cdot \varepsilon & 1 \end{pmatrix}^T \quad Z := \text{polyracines}(v) \quad y_s := \sqrt{\frac{4 \cdot Z}{3 \cdot \beta_1}} \quad x_s := \arccos \left[\frac{1}{a_1} \cdot \left(\varepsilon \cdot y_s - \frac{3}{4} \cdot \beta_1 \cdot y_s^3 \right) \right]$$

$$\frac{Z}{h} = \begin{pmatrix} 0.484 \\ 2.135 \\ 2.981 \end{pmatrix} \quad y_s = \begin{pmatrix} 1.51 \\ 3.173 \\ 3.749 \end{pmatrix} \quad x_s = \begin{pmatrix} 0.408 \\ 0.984 \\ 1.75 \end{pmatrix} \quad D(t, Y) := \left[\begin{array}{l} \left(\frac{-h}{2} \cdot Y_0 + \frac{a_1}{2} \cdot \sin(Y_1) \right) \lambda \\ \lambda \cdot \left[\frac{-\varepsilon}{2} + \frac{3}{8} \cdot \beta_1 \cdot (Y_0)^2 + \frac{a_1}{2 \cdot Y_0} \cdot \cos(Y_1) \right] \end{array} \right]$$

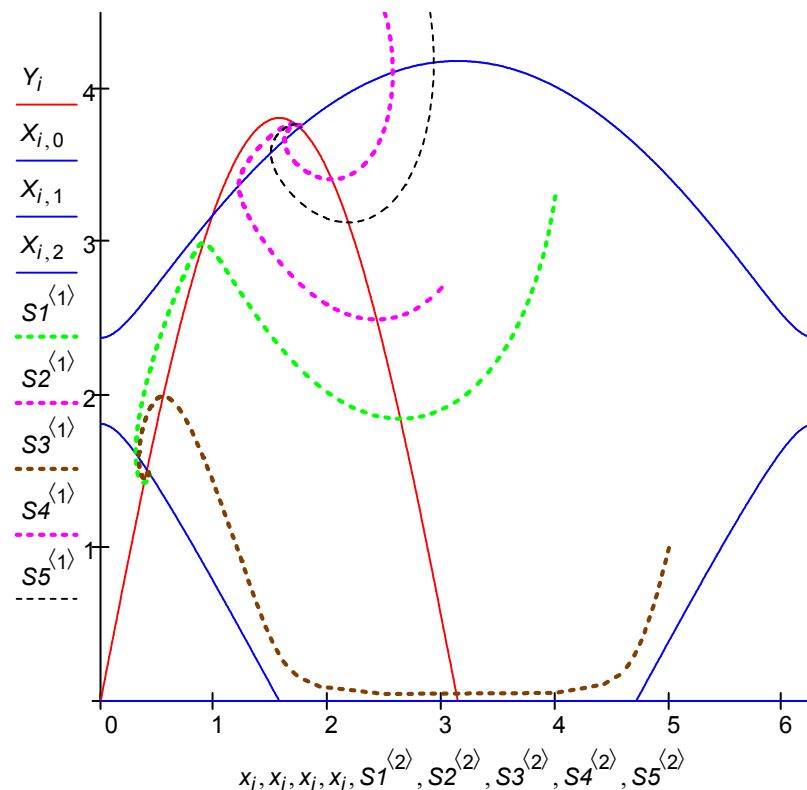
$$t_f := 4000 \quad Y0 := (3.3 \quad 4)^T \quad S1 := \text{rkfixe}(Y0, 0, t_f, n, D) \quad S1^{(2)} := \text{mod}(S1^{(2)}, 2 \cdot \pi)$$

$$Y0 := (4.5 \quad 2.5)^T \quad S2 := \text{rkfixe}(Y0, 0, t_f, n, D) \quad Y0 := (1 \quad 5)^T \quad S3 := \text{rkfixe}(Y0, 0, t_f, n, D)$$

$$S2^{(2)} := \text{mod}(S2^{(2)}, 2 \cdot \pi) \quad S3^{(2)} := \text{mod}(S3^{(2)}, 2 \cdot \pi)$$

$$Y0 := (2.7 \quad 3)^T \quad S4 := \text{rkfixe}(Y0, 0, t_f, n, D) \quad Y0 := (4.5 \quad 2.9)^T \quad S5 := \text{rkfixe}(Y0, 0, t_f, n, D)$$

$$S4^{(2)} := \text{mod}(S4^{(2)}, 2 \cdot \pi) \quad S5^{(2)} := \text{mod}(S5^{(2)}, 2 \cdot \pi)$$



Régimes transitoires vers un foyer stable et un noeud structurellement instable

Excitation harmonique

$$A := 2 \cdot A_0$$

$$A = 0.029$$

$$\varepsilon := 3.16 \cdot h$$

$$\varepsilon = 0.671$$

$$a_1 := \sqrt{\frac{4 \cdot A}{3 \cdot \beta_1}}$$

$$F_{\text{harm}} := a_1 \cdot (\lambda \cdot J \cdot \omega_0^2)$$

$$F_{\text{harm}} = 1.203 \times 10^{-5} \text{ N} \cdot \text{m}$$

$$\sqrt{3} = 1.732$$

$$Y_i := \frac{a_1}{h} \cdot \sin(x_i) \cdot (0 < x_i < \pi)$$

$$X := \left[\begin{array}{l} \text{for } i \in 0..n \\ \left| \begin{array}{l} Z \leftarrow \text{polyracines} \left(\begin{pmatrix} -a_1 \cdot \cos(x_i) & \varepsilon & 0 & \frac{-3}{4} \cdot \beta_1 \end{pmatrix}^T \right) \\ \text{for } j \in 0..2 \\ X_{i,j} \leftarrow Z_j \cdot (\text{Im}(Z_j) = 0) \cdot (\text{Re}(Z_j) > 0) \end{array} \right. \\ X \end{array} \right]$$

$$v := \begin{pmatrix} -A & \varepsilon^2 + h^2 & -2 \cdot \varepsilon & 1 \end{pmatrix}^T \quad Z := \text{polyracines}(v) \quad y_s := \sqrt{\frac{4 \cdot Z}{3 \cdot \beta_1}} \quad x_s := \arccos \left[\frac{1}{a_1} \cdot \left(\varepsilon \cdot y_s - \frac{3}{4} \cdot \beta_1 \cdot y_s^3 \right) \right]$$

$$\frac{Z}{h} = \begin{pmatrix} 0.345 \\ 2.955 \\ 3.02 \end{pmatrix} \quad y_s = \begin{pmatrix} 1.276 \\ 3.733 \\ 3.774 \end{pmatrix} \quad x_s = \begin{pmatrix} 0.341 \\ 1.368 \\ 1.432 \end{pmatrix} \quad D(t, Y) := \begin{pmatrix} \left(\frac{-h}{2} \cdot Y_0 + \frac{a_1}{2} \cdot \sin(Y_1) \right) \lambda \\ \lambda \cdot \left[\frac{-\varepsilon}{2} + \frac{3}{8} \cdot \beta_1 \cdot (Y_0)^2 + \frac{a_1}{2 \cdot Y_0} \cdot \cos(Y_1) \right] \end{pmatrix}$$

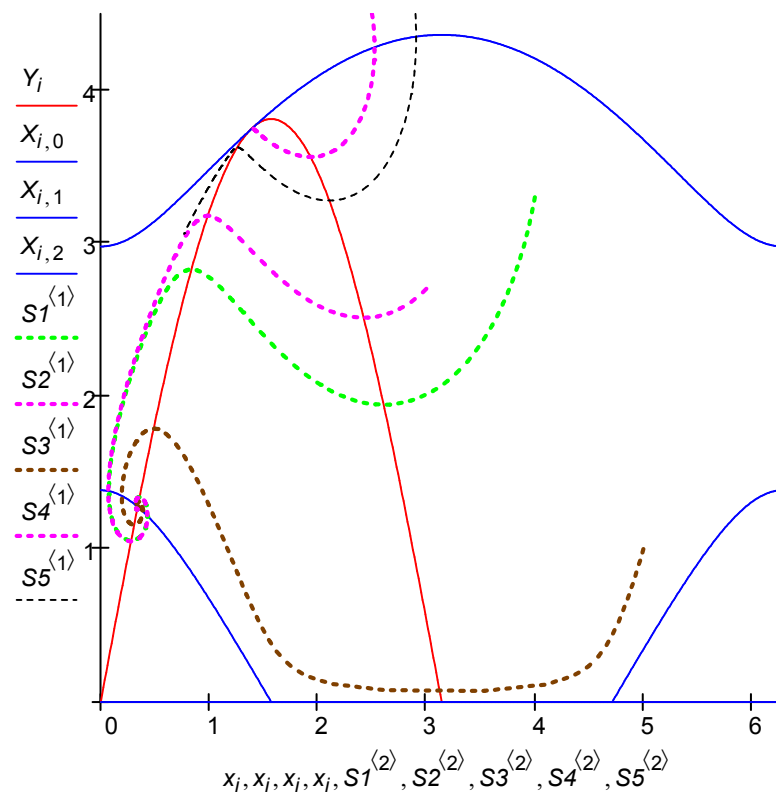
$$t_f := 4000 \quad Y_0 := (3.3 \ 4)^T \quad S1 := \text{rkfixe}(Y_0, 0, t_f, n, D) \quad S1^{(2)} := \text{mod}(S1^{(2)}, 2 \cdot \pi)$$

$$Y_0 := (4.5 \ 2.5)^T \quad S2 := \text{rkfixe}(Y_0, 0, t_f, n, D) \quad Y_0 := (1 \ 5)^T \quad S3 := \text{rkfixe}(Y_0, 0, t_f, n, D)$$

$$S2^{(2)} := \text{mod}(S2^{(2)}, 2 \cdot \pi) \quad S3^{(2)} := \text{mod}(S3^{(2)}, 2 \cdot \pi)$$

$$Y_0 := (2.7 \ 3)^T \quad S4 := \text{rkfixe}(Y_0, 0, t_f, n, D) \quad Y_0 := (4.5 \ 2.9)^T \quad S5 := \text{rkfixe}(Y_0, 0, t_f, n, D)$$

$$S4^{(2)} := \text{mod}(S4^{(2)}, 2 \cdot \pi) \quad S5^{(2)} := \text{mod}(S5^{(2)}, 2 \cdot \pi)$$



Régimes transitoires vers un foyer unique de faible amplitude

Excitation harmonique

$$A := 2 \cdot A_0$$

$$A = 0.029$$

$$\varepsilon := 3.45 \cdot h$$

$$\varepsilon = 0.732$$

$$a_1 := \sqrt{\frac{4 \cdot A}{3 \cdot \beta_1}}$$

$$F_{\text{harm}} := a_1 \cdot (\lambda \cdot J \cdot \omega_0^2)$$

$$F_{\text{harm}} = 1.203 \times 10^{-5} \text{ N} \cdot \text{m}$$

$$\sqrt{3} = 1.732$$

$$Y_i := \frac{a_1}{h} \cdot \sin(x_i) \cdot (0 < x_i < \pi)$$

$$X := \begin{bmatrix} \text{for } i \in 0..n \\ Z \leftarrow \text{polyracines} \left(\begin{pmatrix} -a_1 \cdot \cos(x_i) & \varepsilon & 0 & \frac{-3}{4} \cdot \beta_1 \end{pmatrix}^T \right) \\ \text{for } j \in 0..2 \\ X_{i,j} \leftarrow Z_j \cdot (\text{Im}(Z_j) = 0) \cdot (\text{Re}(Z_j) > 0) \\ X \end{bmatrix}$$

$$v := \begin{pmatrix} -A & \varepsilon^2 + h^2 & -2 \cdot \varepsilon & 1 \end{pmatrix}^T \quad Z := \text{polyracines}(v) \quad y_s := \sqrt{\frac{4 \cdot Z}{3 \cdot \beta_1}} \quad x_s := \arccos \left[\frac{1}{a_1} \cdot \left(\varepsilon \cdot y_s - \frac{3}{4} \cdot \beta_1 \cdot y_s^3 \right) \right]$$

$$\frac{Z_0}{h} = 0.278 \quad y_{s_0} = 1.146 \quad x_{s_0} = 0.305 \quad D(t, Y) := \begin{bmatrix} \left(\frac{-h}{2} \cdot Y_0 + \frac{a_1}{2} \cdot \sin(Y_1) \right) \lambda \\ \lambda \cdot \left[\frac{-\varepsilon}{2} + \frac{3}{8} \cdot \beta_1 \cdot (Y_0)^2 + \frac{a_1}{2 \cdot Y_0} \cdot \cos(Y_1) \right] \end{bmatrix}$$

$$t_f := 4000 \quad Y_0 := (3.3 \ 4)^T \quad S1 := \text{rkfixe}(Y_0, 0, t_f, n, D) \quad S1^{(2)} := \text{mod}(S1^{(2)}, 2 \cdot \pi)$$

$$Y_0 := (1.5 \ 0)^T \quad S2 := \text{rkfixe}(Y_0, 0, t_f, n, D) \quad Y_0 := (1 \ 5)^T \quad S3 := \text{rkfixe}(Y_0, 0, t_f, n, D)$$

$$S2^{(2)} := \text{mod}(S2^{(2)}, 2 \cdot \pi) \quad S3^{(2)} := \text{mod}(S3^{(2)}, 2 \cdot \pi)$$

$$Y_0 := (2.7 \ 3)^T \quad S4 := \text{rkfixe}(Y_0, 0, t_f, n, D) \quad Y_0 := (4.5 \ 3)^T \quad S5 := \text{rkfixe}(Y_0, 0, t_f, n, D)$$

$$S4^{(2)} := \text{mod}(S4^{(2)}, 2 \cdot \pi) \quad S5^{(2)} := \text{mod}(S5^{(2)}, 2 \cdot \pi)$$

